REAL-TIME FUZZY LOGIC COMPUTATIONS

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ABSTRACT

Traditionally digital systems and electronics have been based on Boolean algebra and binary numbers. It is, however, possible to utilize fuzzy set theory in digital applications, especially for control applications in consumer electronics and other real-time systems. Fuzzy logic has been used effectively to handle nonlinear systems. In order to accelerate the operation of fuzzy logic systems we have developed hardware chips that allow direct computation and manipulation of fuzzy set membership functions, fuzzy set operators, fuzzification, fuzzy associative matrices, and center of area defuzzification. Unlike software implementations of fuzzy set theory, the computational architecture we have designed allows all fuzzy logic computations to be performed completely in parallel and in real-time.

Keywords

Fuzzy Logic, Fuzzy Set Theory, Control Systems

INTRODUCTION

Traditional electrical and mechanical control systems are based primarily on systems of differential equations. Nonlinearities in the systems have always been a problem. One solution is to make linear approximations for a specific operating point. However, the complexity and performance demands of today's systems are making this increasingly difficult to accomplish. Another approach has been to digitize the systems and use various digital signal processing techniques. These tend to be computationally intensive solutions and performance depends on the sampling frequency of the system as well as the computational abilities of the processors used to perform the Fast Fourier Transform calculations that often involve floating-point calculations of complex numbers.

A separate train of thought for control systems has been developing for a number of years based on fuzzy logic and fuzzy set theory developed by Zadeh

(Zadeh, et. al. 1992). The fuzzy approach to linear and nonlinear control systems eliminates the use of differential equations and transforms the problem into a largely algebraic problem. Most fuzzy controllers developed up to this point have all been implemented as software programs on standard computing or embedded systems platforms. Instead of using typical linear programming methods, we have developed a custom processor architecture that performs the fuzzy computations and control in a parallelized method.

One of the key concepts in fuzzy systems is to determine the degree of membership a variable has to various elements of a fuzzy set. In a traditional Boolean set the membership or nonmembership of a variable to a particular portion of the set is described by a characteristic function $\mu_A(x)$, where

$$\mu_A(x) = 1, \text{ if } x \in A, \text{ and}$$
$$\mu_A(x) = 0, \text{ if } x \notin A.$$

In a fuzzy set, however, partial memberships are possible, and the value of membership is a real valued number between 0 and 1 inclusive, i.e.

$$\mu_A(x) = [0.0,\ 1.0].$$

The distinction between Boolean and fuzzy sets can be more easily understood with Figures 1 through 3. Figure 1 illustrates that all values of height which are less than 182.5 cm have a degree of membership of 1.0 to the category "Short" and no membership to the category "Tall". Heights greater than 182.5 cm have no membership to category Short and full membership to category Tall. Thus, a person with a height of 182 cm is classified as 100% Short and 0% Tall. Similarly, a person with a height of 183 cm is 100% Tall and 0% Short. However, in the fuzzy system of Figure 2 a person with height 183 cm is both Short and Tall. Figure 3 shows more specifically that such a person would be classified as 0.4 Short and 0.6 Tall. The triangular membership functions shown are rather arbitrary. Other types of fuzzy membership functions such as trapezoids can be used. The particular shapes, slopes, and ranges of the fuzzy membership functions have to be determined by an expert which is why fuzzy logic is generally considered to be part of the field of artificial intelligence.

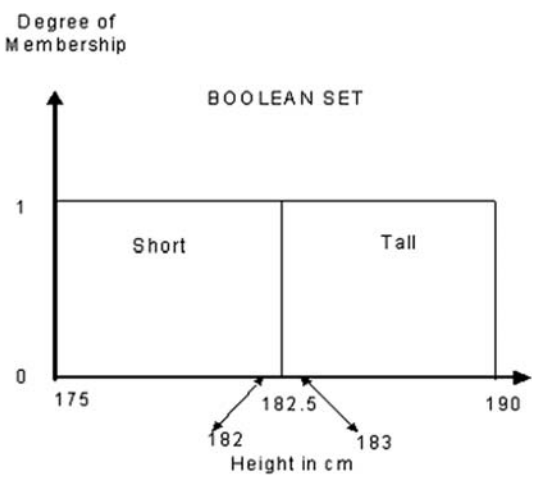

Figure 1. Graphical representation of the membership functions of a Boolean set.

METHODS

The fuzzy control approach generally has three stages: fuzzification, rule evaluation, and defuzzification. For fuzzification, one or more "crisp" inputs are sampled by the system and then categorized or fuzzified using the various fuzzy membership functions to describe how much the input belongs to each element of the system's fuzzy set. An example application would be a greenhouse humidity control system. In this case, the actual relative humidity could be measured as well as the current difference between actual and desired humidity. Figure 4 shows the five membership functions that apply to the fuzzification of relative humidity. The sample data point illustrates that 64% relative humidity gets fuzzified as 0.5 Humid and 0.7 Normal (it should be noted that it is not necessary for the sum of memberships to be 1.0). Figure 5 shows the five membership functions for error in relative humidity and that a humidity error of –4% is fuzzified as 0.4 Negative Low and 0.6 None.

Based on the current inputs' degrees of membership in each of the fuzzy set members, a series of rules is evaluated to determine what the fuzzy output should be. Sometimes the

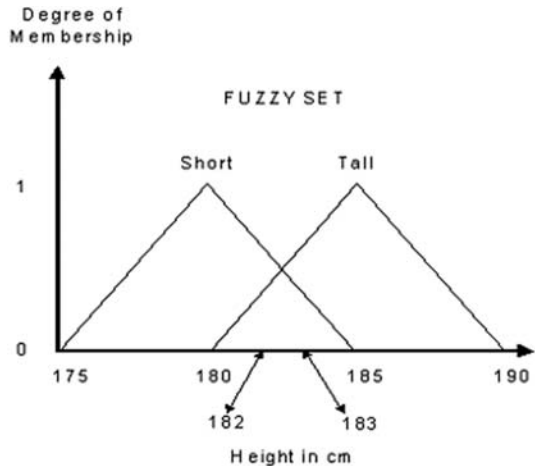

Figure 2. Graphical representation of the membership functions of a fuzzy set.

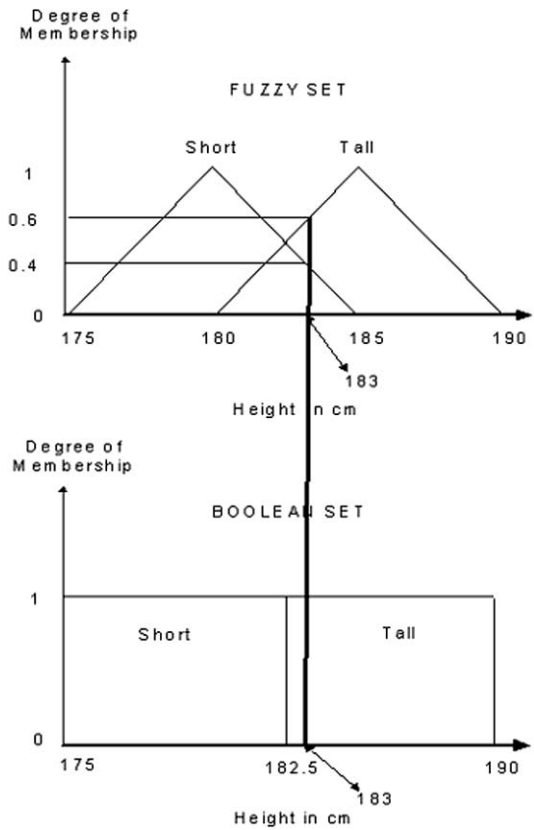

Figure 3. Graphical comparison of the classification of height in a Boolean set to the fuzzification of height in a fuzzy set.

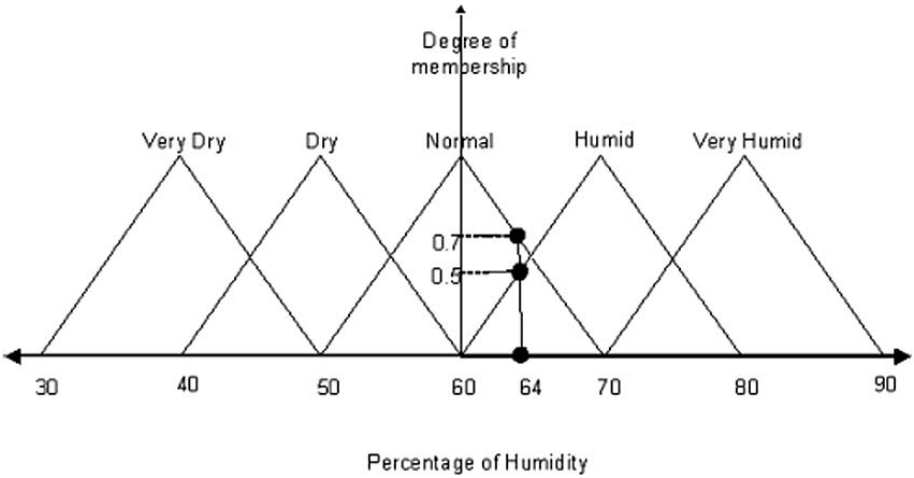

Figure 4. Graphical representation of the five membership functions for the crisp input of relative humidity. Shown is the fuzzification of a sample input value of 64% relative humidity into the two membership functions Normal and Humid. Here, the value of relative humidity fuzzifies into 0.7 Normal and 0.5 Humid.

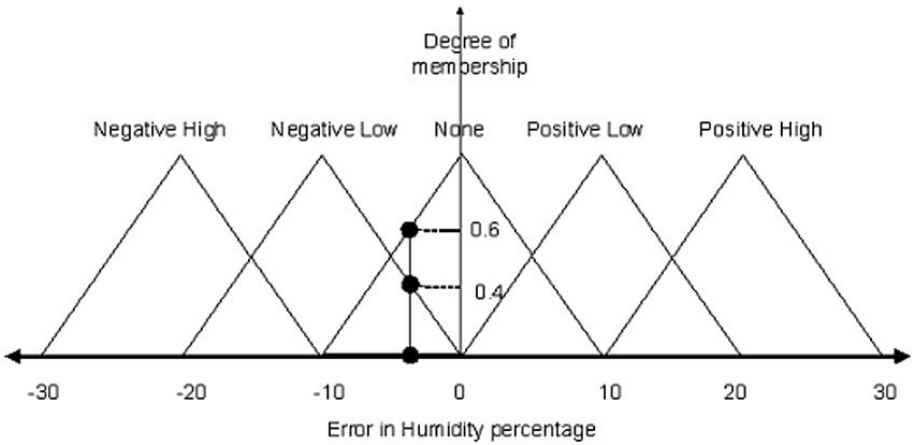

Figure 5. Graphical representation of the five membership functions for the crisp input of error in relative humidity. Shown is the fuzzification of a sample input value of –4% error in relative humidity into the two membership functions Negative Low and None. Here, the value of error in relative humidity fuzzifies into 0.4 Negative Low and 0.6 None.

rule evaluation is viewed as a series of IF-THEN statements similar to what are used in expert systems. However, it is more common to summarize the rules in a fuzzy associative matrix as seen in Figure 6. The evaluation of a rule in the fuzzy associative matrix leads to the generation of a fuzzy output. A rule evaluation can occur at every intersection of row and column. Looking at Figure 6, if the humidity is Dry and the error in humidity is Negative Low then there should be a Highly Positive output (the output being the injection of water mist into the greenhouse). Or for example, if the humidity is Normal and the error in humidity is None then the output should be Zero. We arbitrarily choose the numerical value of fuzzy output membership to be the minimum of the degree of memberships of the fuzzified inputs as illustrated in Figure 7. Since the input(s) can belong to one or more membership functions, usually several rules in the fuzzy associative matrix are triggered and have to be evaluated and thus lead to several simultaneous fuzzy outputs. In this example the humidity is both Humid and Normal and the error in humidity is both Negative Low and None so four rules in the

	Neg. High	Neg. Low	None	Pos. Low	Pos. High
Very Dry	PH	PH	PH	PL	ZE
Dry	PH	PH	PL	ZE	NL
Normal	PH	PL	ZE	NL	NH
Humid	PL	ZE	NL	NL	NH
Very Humid	ZE	NL	NH	NH	NH

Figure 6. The fuzzy associative matrix for rule evaluation in the humidity control example. The rows of the matrix correspond to the membership functions for fuzzified relative humidity. The columns of the matrix correspond the membership functions for fuzzified error in relative humidity. The elements of the matrix are the fuzzy outputs. The various fuzzy outputs are: PH – Positive High, PL – Positive Low, ZE – Zero, NL – Negative Low, and NH – Negative High.

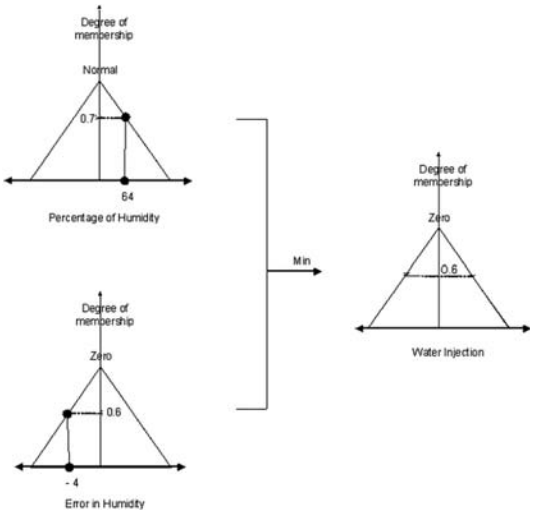

Figure 7. Graphical representation of how the rule evaluation for two fuzzified inputs maps to the corresponding value of fuzzy output. Here, the combination of Normal relative humidity and None error in relative humidity evaluates to a value of 0.6 in the fuzzy output membership function of Zero.

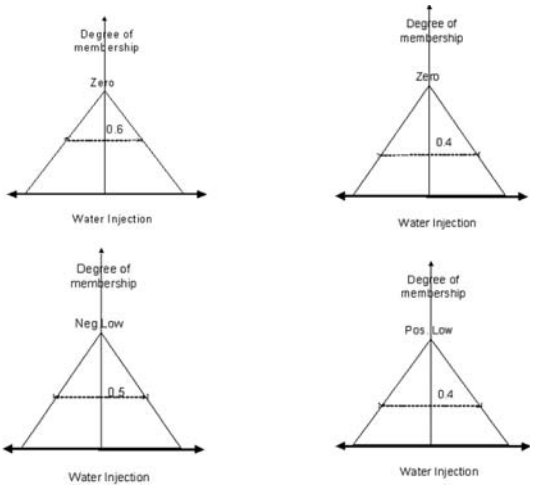

Figure 8. Graphical representation of the four fuzzy output memberships obtained after rule evaluation for sample inputs of 64% relative humidity and –4% error in relative humidity.

fuzzy associative matrix are triggered and result in the four fuzzy output memberships shown in Figure 8.

The various fuzzy outputs have to be combined into a single "crisp" output (occasionally, multiple crisp outputs are created for certain systems using the same crisp inputs). This conversion is called the defuzzification process. Several techniques have been developed to calculate the defuzzified output. We have chosen to use the Center of Area method where the defuzzified output is found by

$$Output = \frac{\sum_{i=1}^{N} A_i * C_i}{\sum_{i=1}^{N} A_i} \quad \text{(Equation 1)}$$

where N is the number of fuzzy output membership functions, A_i is the active area of each output membership function triggered by the rule evaluation, and C_i is the center of the output membership function. The application of Equa-

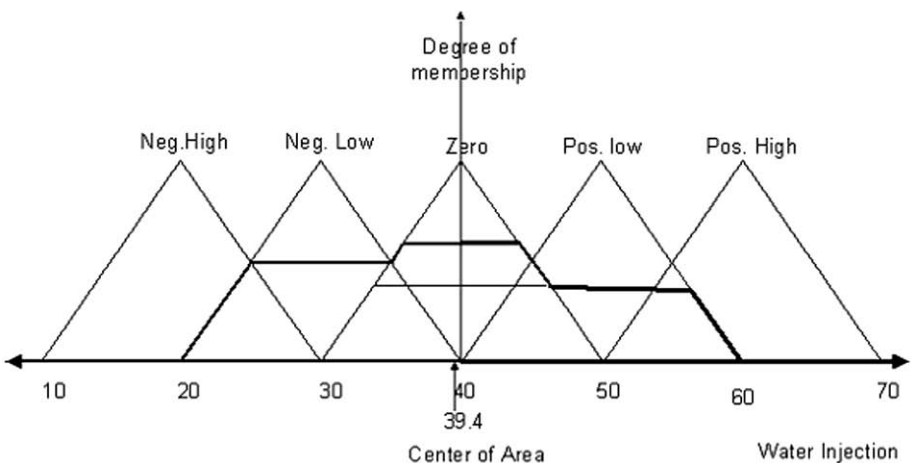

Figure 9. Graphical representation of defuzzifying the four fuzzy output memberships in the humidity control example. The center of area for the four fuzzy outputs from Figure 8 is a crisp output value of 39.4 for the water injection level.

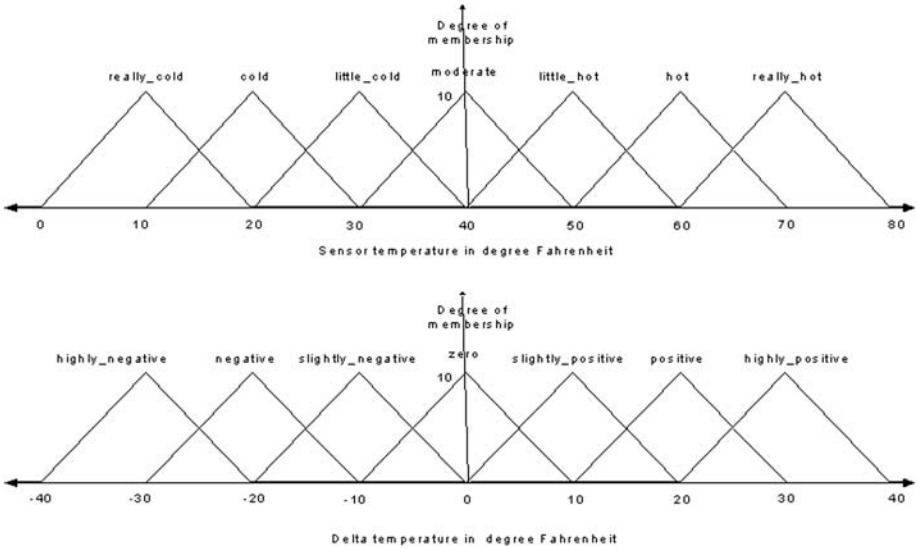

Figure 10. Graphical representation of the seven membership functions for each of the crisp inputs of temperature and error in temperature.

tion 1 to the humidity example results in a water injection output with a crisp value of 39.4 as illustrated graphically in Figure 9.

To demonstrate the ability to parallelize the fuzzy operations in hardware we have implemented a temperature control system using the membership functions shown in Figure 10 for fuzzification, the fuzzy associative matrix shown in Figure 11 for rule evaluation, and the Center of Area method for defuzzification. The actual defuzzified output goes to a separate motor interface circuit to regulate the fan speed on a ventilation system. The details of the arithmetic circuitry and the motor interface will be dis-

	HN	N	SN	ZE	SP	P	HP
Real Cold	VC	VC	VC	NO	LH	H	VH
Cold	VC	VC	C	NO	LH	H	VH
Little Cold	VC	VC	C	NO	LH	H	VH
Mod erate	VC	C	C	NO	H	H	VH
Little Hot	VC	C	LC	NO	H	VH	VH
Hot	VC	C	LC	NO	H	VH	VH
Real Hot	VC	C	LC	NO	VH	VH	VH

Figure 11. The fuzzy associative matrix for rule evaluation in the temperatuve control system. The rows of the matrix correspond to the membership functions for fuzzified temperature. The columns of the matrix correspond the membership functions for fuzzified error in temperature. These values are: HN – Highly Negative, N – Negative, SN – Slightly Negative, ZE – Zero, SP – Slightly Positive, P – Positive, and HP – Highly Positive. The elements of the matrix are the fuzzy outputs. The various fuzzy outputs are: VC – Very Cold Air, C – Cold Air, LC – Little Cold Air, NO – No operation, LH – Little Hot Air, H – Hot Air, VH – Very Hot Air.

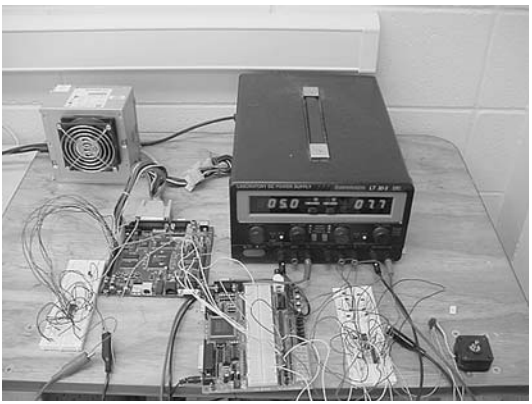

Figure 12. Experimental setup of prototype temperature controller with test equipment.

cussed elsewhere (Hemmelman, et. al., 2003). The prototype circuit is shown in Figure 12.

RESULTS

A custom fuzzy processor designed specifically for parallelizing the calculations required for fuzzification, rule evaluation, and defuzzification has been designed and implemented in a Field Programmable Gate Array (FPGA). The fuzzy set membership functions, the fuzzy associative matrix, and the Center of Area formula for the temperature control system were all successful hardwired into the processor. Testing of the chip on various inputs has demonstrated that the proper motor interface output signal is generated by the processor. The delay time for the fuzzy computations is strictly a matter of combinational logic propagation delay so real-time control is readily achievable.

CONCLUSIONS

Fuzzy logic and fuzzy set operations have been successfully hardwired into a custom processor that parallelizes the calculations allowing for real-time operation of a control system. This has been demonstrated in a temperature control application. However, the fuzzy processor design is suitable for any fuzzy control application.

LITERATURE CITED

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