THE SHAPE OF CELESTIAL OBJECTS

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ABSTRACT

Planets and larger natural satellites in the Solar System are nearly spheroidal in shape, but smaller satellites and all but the largest asteroids tend to be ellipsoids with varying lengths of the three major axes. A comparison of ellipsoidicity and size shows that celestial objects bigger than approximately 1,000 km in diameter are spheroidal. This is not necessarily only because the large objects are (or were) liquid, but is because of surface modification by meteor impacts and gravitationally-induced mass-wasting processes. Meteoritic bombardment changes the shape of a celestial object, and it becomes spheroidal, the highest entropy state for a body subject only to self-gravitation.

Keywords

Celestial objects, ellipsoidicity, entropy, impact craters

INTRODUCTION

As telescopes and space probes become more refined, it is possible to better quantify the shape of planets, natural satellites, and asteroids. Not all celestial objects in the Solar System are spherical. Smaller ones are quite angular and ellipsoidal. In this paper we examine the shape of celestial objects with respect to their size, and offer a hypothesis to help explain the relationship between ellipsoidicity and size.

The working hypothesis for this paper is that processes such as meteor impact and mass wasting act on a celestial object and eventually change an irregularly-shaped object into a spherical shape. This process is more effective on a celestial object with sufficient mass to cause debris displaced by meteor impacts to be returned to the surface. Higher gravity also allows for gravity-induced geomorphologic processes, such as rockfall, creep, and landslide, to be more effective. The rugged topography, as well as the presence of surficial deposits on inclined slopes on the surface of an irregular-shaped object with a high ellipsoidicity has higher potential energy than a spheroidal-shaped object. The higher the value of ellipsoidicity, the higher the energy state for this celestial object. Another way of stating this is that a solid celestial object of sufficient mass to facilitate large gravitational forces will eventually assume a spheroidal shape since this is the highest entropy. Celestial landscapes, like terrestrial landscapes, evolve towards increased entropy (Leopold and Langbein 1962): "The distribution of energy to a geomorphic system… will, in the process of landscape evolution, move downhill."

Satellite shapes. To describe the shape of sedimentary particles, geologists commonly use the term "sphericity"; this term utilizes non-parametric ratings from one to ten (Krumbein and Sloss 1951). In this paper, however, the parametric term "ellipisoidicity" is used to describe shape. This term uses a mathematical formula involving the ratios of the three major axes (Table 1). The formula yields a value of unity for a perfect sphere.

Table 1. Satellite diameter and ellipsoidicity. Data for three major axes are from Menzel and Pasachoff (1983) supplemented by recent spacecraft observations. Ellipsoidicity values calculated by the authors.

	Satellite Axes	Diameter (km)	Ellipsoidicity*
Earth	The Moon	3,476	~1.00
Mars	Phobos	27 x 21 x 18	1.93
	Deimos	15 x 12 x 10	1.88
Jupiter	Amalthea	270 x 165 x 153	2.89
	Io	3,632	~1.00
	Europa	3,126	~1.00
	Ganymede	5,276	~1.00
	Callisto	4,820	~1.00
	Thebe	270 X 165 X 158	2.80
Saturn	Atlas	80 x 60 x 40	2.67
	Janus	220 x 200 x 160	1.51
	Epimetheus	140 X 120 X 106	1.54
	Mimas	390	~1.00
	Titan	5,150	~1.00
	Hyperion	400 x 250 x 220	2.91
Uranus	Miranda	600	~1.00
Neptune	Triton	3,200	~1.00
Pluto	Charon	1,214	~1.00

* "Ellipsoidicity", the departure from a perfect sphere, is defined as follows (modified from Krumbein, 1941). Consider an object with a crudely ellipsoidal shape with diameters of its three major axes: long (D_a), intermediate (D_b), and short (D_c). Ellipsoidicity (Ell.) is defined as: Ell. = D_a/D_b x D_a/ D_c. A perfect sphere would have equal lengths of all three axes; hence: Ell. =1.000.

Table 1 shows the size and ellipsoidicity of some of the largest satellites in the Solar System. Jupiter's four major satellites (Io, Europa, Ganymede, and Callisto) are spherical (ellipsoidicity is ~1). They are approximately the size of Earth's Moon, but are small compared to Jupiter itself (Figure 1). NASA's spacecraft *Galileo* made a close flyby of Io in 2000; Io has an estimated 300 volcanoes erupting at any given time (Benson 2003). Io has few surface impact craters because subsequent volcanic activity has obscured them. In this sense, Io has the youngest surface in the Solar System. Europa, also spherical, likely has a thin ice crust over a global ocean; although it is cold on the outside, it seems to be liquid at depth. In contrast to Jupiter's four major satellites, Thebe, one of Jupiter's smaller satellites, has a relatively high ellipsoidicity value of 2.8 (Table 1).

Table 1 shows data for six of Saturn's satellites. The four smallest satellites are ellipsoidal in shape.

Mars has two very small satellites: Phobos and Deimos. These small satellites are quite ellipsoidal.

Five satellites orbit the dwarf planet Pluto. The *NewHorizon* spacecraft mission flyby confirmed four small satellites (equivalent diameter ~40 km) are highly elongated compared to "…Charon, which is nearly spherical with a diameter of 1,220 km" (Weaver et al. 2016).

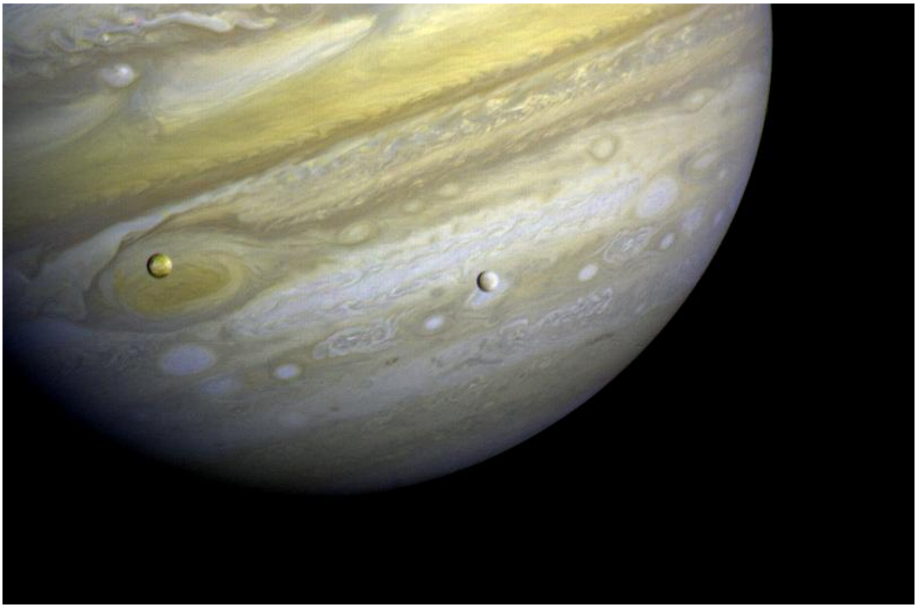

Figure 1.* Voyager 1 *spacecraft image of Io and Europa in front of Jupiter. Io appears as a tiny red sphere suspended in front of Jupiter's giant red spot, and Europa is at the far right. From: http://www.jpl.nasa.gov/spaceimages/details.php?id=PIA00144

Asteroid shapes. Table 2 shows the size and ellipsoidicity of some of the known asteroids. Within the main asteroid belt, approximately 200 asteroids have been discovered with diameters exceeding 100 km (Price, 2001). The diameter of asteroids can be roughly determined by the time it takes to pass in front of ("occult") a star.

Some space missions have passed close to a few asteroids, providing more definitive data concerning their shape. NASA's *Dawn* spacecraft took images of Ceres that show the meteor crater Occator in detail (Figure 2). Ceres is the largest known asteroid and is nearly spheroidal (ellipsoidicity equals 1.12). Other large asteroids such as Vesta and Pallas are also nearly spherical.

In 1997, a Near Earth Asteroid Rendezvous (NEAR) spacecraft took several hundred images of the asteroid Mathilde showing it to be fairly spherical, even though it has a diameter of only 53 km (Thomas et al. 1999).

In 2001, the NEAR spacecraft *Shoemaker* surveyed and landed on the small, dust-covered, potato-shaped asteroid named Eros (Head 2000; Check 2001). Eros (Figure 3) has a high ellipsoidicity value of 3.13.

Table 2. Size and ellipsoidicity of some of the brightest asteroids. Major axes data from Menzel and Pasachoff (1983) supplemented by spacecraft observations. Ellipsoidicity values calculated by the authors; where data for only two axes are available it is assumed that the two minor axes have equal diameters.

Name	Axes Diameter (km)	Ellipsoidicity
Vesta	580 X 530 X 470	1.33
Pallas	583 X 532 X 536	1.21
Ceres	959 X 907	1.12
Iris	222	
Eros	23 X 13 X 13	3.13
Hebe	206	
Juno	288 X 230	1.57
Melpomene	164	
Eunomia	261	
Flora	160	
Bauberga	256	
Ganymed	40	
Metis	168	
Nausikaa	99	
Massalia	140	
Mathilde	60	

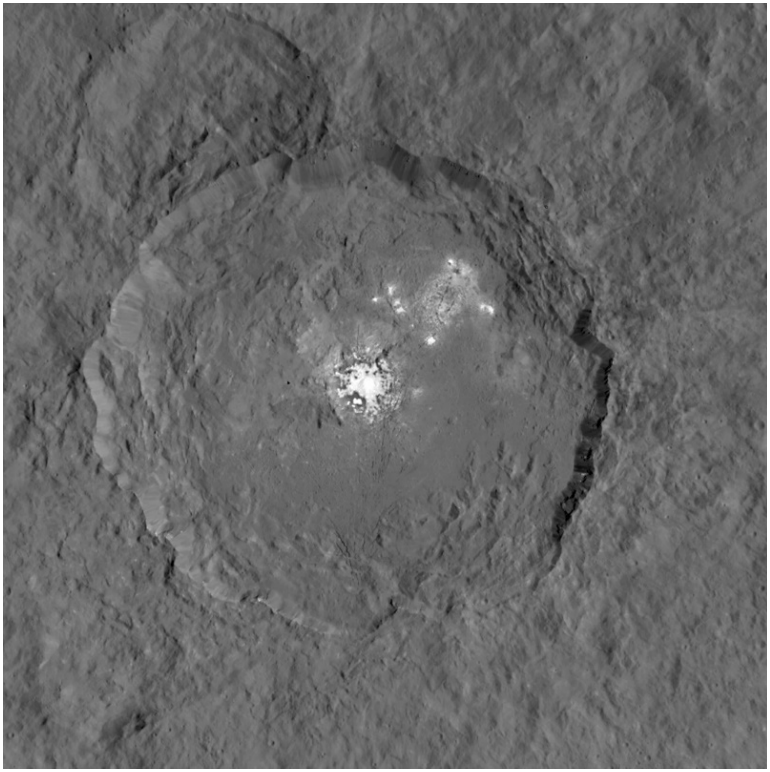

Figure 2. Meteor crater Occator on asteroid Ceres. Landslides and talus are present along the crater walls. From: https://www.nasa.gov/image-feature/jpl/dawn-takes-a-closer-look-at-occator

Figure 3. The potato-shaped asteroid Eros image generated by data from the laser rangefinder on NASA's "Near Earth Asteroid Rendezvous – Shoemaker" (NEAR Shoemaker) spacecraft. The image is posted on NASA's website at: https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=2061

Meteor craters. Asteroids have been impacting the surface of celestial objects in the Solar System for billions of years. The asteroid belt lies primarily between Mars and Jupiter. The size distribution of asteroids that originated from fragmentation of a parent body is discussed by Tanga et al. (1999).

There are approximately 150 discernible meteor impact structures on Earth (Grieve et al. 1995). Brecciated and fractured rocks surrounding craters are described by Zenchenko and Tsvetkov (1999). Breccia zones and melted zones of the 35.7 Ma Popigai Crater, Russia, were mapped by Masaitis et al. (1999). Ejected debris from the Chicxulub impact (dated at the K-T boundary) fell back into the atmosphere around the entire globe (Kring 2000). Cratering and impact debris are well known at Meteor Crater, Arizona, and Alamo, Nevada. Earth's most famous meteor impact structure is Meteor (Barringer) Crater, which formed from a meteor impact approximately 50,000 ago. Due to Earth's hydrosphere, atmosphere, and active geologic activity, geomorphic processes on Earth proceed rapidly compared to the Moon; hence Earth has fewer surface impact craters that are still discernable.

The craters of the Moon were once thought to be volcanic in origin but Gilbert (1893) showed that they were caused by meteor impacts. Surface modification on the Moon by meteor impact, as well as modification by mass-wasting processes and volcanism, is well documented (Arvidson et al. 1975; Shoemaker et al. 1970; O'Keefe and Ahrens 1977). Sharp lunar crater rims become lowered and older craters become filled with debris (Cannon, 1970). The Apollo missions to the Moon, beginning in 1969, show a cratered surface covered with dust (Figure 4). Ejecta from meteor bombardments and mass-wasting processes modify the surface of the Moon, creating a thick regolith that tends to fill in low places. Most satellites and asteroids have regoliths principally caused by meteor impacts and the resulting ejecta.

Planets other than Earth have also been bombarded by meteors. The *Magellan* probe to Venus (1990-1994) shows a meteor-cratered landscape and extensive lava, somewhat similar to the Moon. Mars and Mercury show that a large percentage of their surfaces are pock-marked by meteor impacts. NASA's *Pathfinder* spacecraft traveled to Mars in 1997. The *Pathfinder* rover called *Sojourner* traversed the surface of Mars finding a regolith with angular rocks on a dusty, wind-swept surface. *Sojourner's* pictures revealed mass-wasting deposits similar to a typical debris-flow deposit on an alluvial fan in the Mojave Desert. Surface modification of craters on Venus by volcanic infilling and mass wasting are described by Wichman (1999). Pluto and its satellite Charon have an impact-dominated surface whose origin may involve mobilization of volatile ice from the interior (Moore et al. 2016).

Szabo and Kiss (2008) studied asteroid brightness and showed how shape evolves from elongated to roughly spherical forms. They believe asteroid shape evolves by meteor impacts causing craters. This is followed by impact breccia being launched and re-impacted on the surface, causing subsequent rearrangement of the regolith. Impacts also generate seismic shaking, causing slumping and downslope movement of the regolith (Kring, 2000). Pullen (2008) notes that "Gyula (Szabo) and his team show that asteroids change shape from elongated to roughly spherical due to being impacted during their lifetimes. They are like pebbles on the beach that become worn smooth over many years—only in space, erosion is caused by small impacts as rocks knock into each other and chip pieces off. Impact specialist Jonti Horner from the UK's Open University agrees with Gyula. 'The results make sense,' he says. 'Catastrophic impacts create a huge slew of fragment shapes, like the shards of a broken bottle. The debris then are weathered over time and smoothed towards sphericality by small impacts.'"

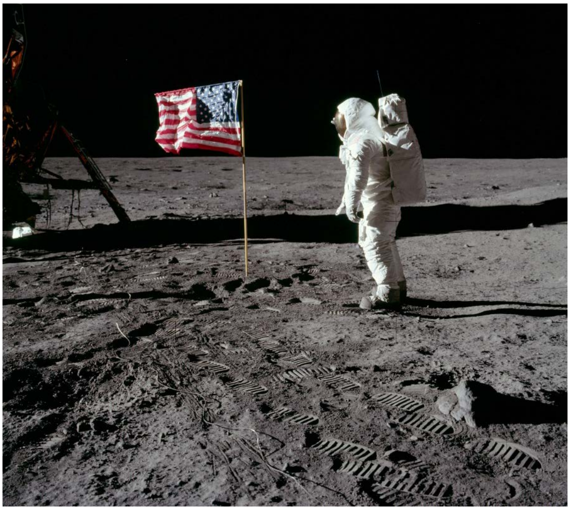

Figure 4. Footprints on the lunar surface made by astronaut Buzz Aldrin and Neil Armstrong during the 1969 Apollo 11 expedition. Little bedrock is exposed since the landscape is covered by dusty meteor ejecta. The regolith is composed of weakly coherent fragmented material (Shoemaker et al. 1970). From: http://www.hq.nasa.gov/alsj/a11/AS11-40-5874HR.jpg

Theory of celestial shapes. Table 3 shows the size of planets in the Solar System. All the planets are nearly spheroidal in shape. Even Pluto (Figure 5), the smallest (~2,370 km diameter), is spherical.

If a celestial object is liquid, it would assume the shape of a sphere due to its own gravity, pulling the liquid surface to the lowest energy state. Planets and satellites also rotate on their axes, and their spheroidal shape becomes modified. Earth's equatorial bulge is well known; technically Earth is an "oblate spheroid". Zeitler and Oberst (1999) describe the influence of the equatorial bulge on the shape of Mars: it has an equatorial radius of 3396 km and a polar radius of 3377 km. [Note: the "ellipsoid" is a mathematical reference surface, whereas the "geoid" is the equipotential surface that sea level follows (Li and Gotze 2001; Urban 2015).]

As a generality, planets and most large satellites are spherical because they were liquid at one time, whereas asteroids are smaller and have irregular (angular) shapes because they were originally a solid body that was broken into pieces. Any rotating liquid celestial object would also develop an equatorial bulge similar to the Earth. Arkani-Hamed et al. (1999) found that the floors of lunar maria tend to lie on a surface in agreement with the equipotential surface that originated

Table 3. Size of planets in the Solar System (Menzel and Pasachoff, 1983).

Planet	Diameter (km)
Mercury	4,878
Venus	12,104
Earth	12,756
Mars	6,794
Jupiter	142,796
Saturn	120,000
Uranus	52,290
Neptune	48,600
Pluto	2,371

from basalt melts that flooded the maria basins. The tremendous energy from massive meteor impacts may locally melt the bedrock and contribute to the evolution of a spherical shape.

However, even a solid celestial object evolves towards a spherical shape. Cataclysmic bombardment forms brecciated ejecta on satellites and asteroids; this ejecta on asteroid Vesta has been dated at 4.56 billion years (Bogard 2005). The crater Occator on the large asteroid Ceres shows evidence of another geomorphic activity: landslides and talus along the crater walls, similar to lunar features (Figure 2). Topographically high areas would be most prone to impact degradation, and, because the celestial object has its own gravity, ejecta would roll, bounce, or slide downgradient and come to rest at a lower place. Changes in temperature facilitate the process of rock disintegration and creep. Seismic shocks from impacting meteors would also facilitate mass-wasting processes.

Figure 6 is a plot of ellipsoidicity vs. size for selected asteroids and satellites. Data are limited, but there is an obvious convergence of shapes towards a sphere as objects become larger. Above 500 km diameter, celestial objects have been modified considerably since ellipsoidicity values are less than 3.0. If a celestial object has a 1,000 km diameter, its ellipsoidicity value is less than 1.5 and the object is nearly spherical.

Because of meteor impacts and geomorphic processes, a large celestial object would tend to become spherical without having been liquid. This is an example of entropy: nature tends to work towards a state of unavailability of energy. As a celestial body is bombarded by meteors, the high areas would be particularly vulnerable to erosion. The ejecta would accumulate in low areas, the place of lowest potential energy. Eventually a celestial object would achieve an "equipotential level," a liquid hydrostatic equilibrium geometry, becoming either a perfect spheroid (or, if rotating, a modified spheroid). Another way to think of this is that a sphere is the most efficient way to contain a given volume.

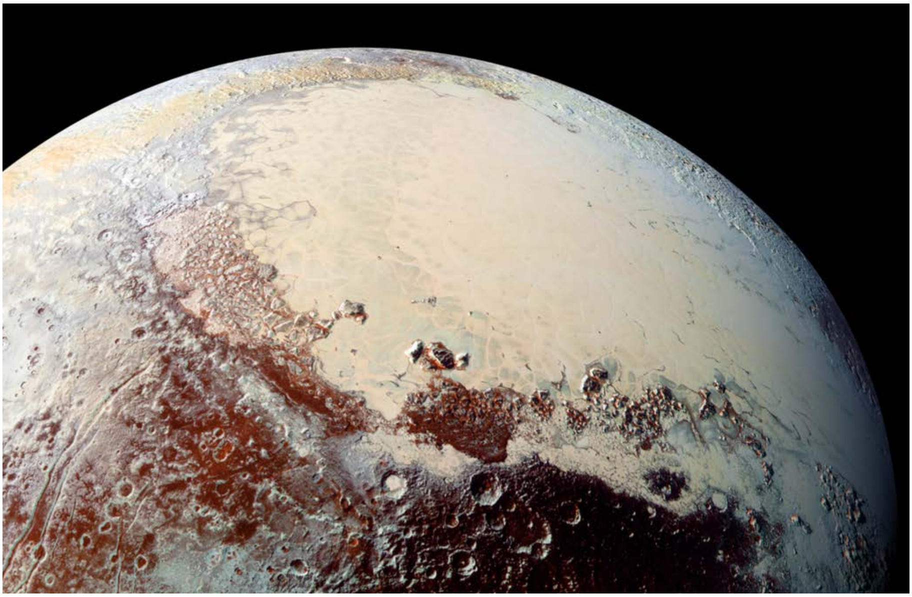

Figure 5. Image of Pluto taken in 2015. The light-colored area is interpreted as geologically-recent ice-flows and the dark area is crater-impacted older terrain. From: http://www.nasa.gov/image-feature/pluto-s-brilliant-heart

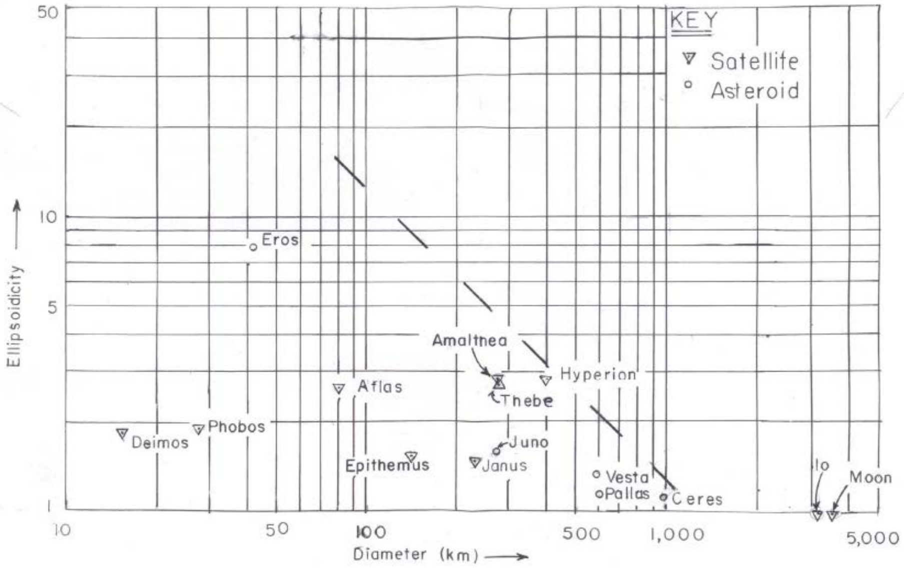

Figure 6. Plot of ellipsoidicity vs. long axis diameter for celestial objects. The dashed line is the approximate limiting ellipsoidicity for any given diameter.

Principle of self-gravitation. Changes in the shape of an asteroid due to collision with meteoroids require that a large percentage of ejecta do not achieve escape velocity (Ronca and Furlong 1979). Ejecta from meteor bombardment may achieve escape velocity and become blasted completely away from a small celestial object (O'Keefe and Ahrens, 1977; Check 2001). Large celestial objects, with larger mass, exact a greater gravitational force on the ejecta, allowing much ejecta to return to the surface. Cunningham (1988) suggests that above 250 km diameter, the mass of an asteroid is sufficient to prevent ejecta fragments from achieving escape velocity.

From basic physics, it can be seen that the gravitational acceleration (a) on its surface depends simply on the celestial object's mass (M) and its radius (R). The general formula for gravitational acceleration is $a = GM/R^2$ where "a" is gravitational acceleration, "G" is the gravitational constant, "M" is the mass of the attracting object, and "R" is the radius from the object center. So, doing the math for Earth, Mars, and the asteroid Ceres, we get the following values for gravitational acceleration: Earth (9.792 m/s^2), Mars (3.73 m/s^2), and Ceres (0.352 m/s^2). Thus, the gravitational acceleration on Ceres is about 3.6% that of the gravitational acceleration on the surface of Earth.

From Figure 6, the diameter of approximately 1,000 km appears to be a critical factor in that objects larger than this tend to be spherical. This is approximately the size of the asteroid Ceres. This suggests that the gravitational acceleration on the surface of a celestial object roughly equal to 3.6% that of Earth seems to be the limiting self-gravitation in order for a celestial object to evolve into a sphere.

SUMMARY

A celestial object is not necessarily spherical because the object was, or is, a liquid. "Self-gravitation" is also an important factor for determining the shape of a celestial object. Available data in the Solar System indicate that celestial objects having a major diameter greater than approximately 1,000 km are nearly spherical. Objects smaller than this have a surface gravity less than 3.6% that of Earth; this is not enough to prevent ejecta from achieving escape velocity. Thus, those ejecta are post to space and do not return. Meteors have bombarded the surface of celestial objects for billions of years, and these impacts change the topography. On a celestial object with a diameter greater than 1,000 km, bombardment of highlands and accumulation of ejecta followed by gravitational mass wasting allow impact ejecta to move into the lowlands. This modifies the topography so as to achieve a spherical shape, as expected according to the laws of fluid mechanics. This modification reduces the overall potential energy of debris on the surface of the celestial object; it evolves towards the most probable state, that of high entropy.

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