AN ANALYTICAL MODEL TO PREDICT THE GEOMETRY OF THE CONE OF DEPRESSION CAUSED BY PUMPING AN ANISOTROPIC AQUIFER

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ABSTRACT

The Madison aquifer in the Black Hills of South Dakota is known for its karst topography, cavernous formations, and anisotropic properties. For example, the Bureau of Land Management (1980) numerically simulated the water withdrawal from this aquifer in the Niobrara well field, Wyoming. The cone of depression around the pumping well created irregularly shaped contours of drawdown. This was primarily due to the directional transmissivity values of the anisotropic Madison aquifer.

Four methods available in the literature for the drawdown distribution around a pumping well in an anisotropic aquifer are: Papadopulos (1965); Hantush and Thomas (1966); Glover and Moody (1976); and Walton (1985). In this paper each method was analyzed by assuming suitable hydrologic parameters for a hypothetical pumping test of the Madison aquifer. The method developed by Hantush and Thomas (1966) was found to be clearest and easiest for deriving the drawdown equation. It also yielded good results.

An analytical model was developed for the Hantush and Thomas (1966) method of analysis of the cone of depression. The computer program was written in FORTRAN, which uses polynomial approximations and the Newton-Raphson iteration procedure to calculate the "inverse well function."

INTRODUCTION

Many aquifers in the Black Hills of South Dakota show anisotropic behavior. The Madison Limestone, the Minnelusa Formation, and the Dakota Sandstone (Rahn, 1992) are examples of aquifers of the Black Hills that have anisotropic properties. Pumping of water from such aquifers creates irregularly shaped contours of drawdown around a pumping well.

The Madison Limestone is South Dakota's most valuable aquifer. Many communuities such as Spearfish, Whitewood, Sturgis, Hot Springs, Edgemont, Philip, and Midland obtain water from the Madison. It is known for its karst topography and cavernous formations. Its permeability depends upon many factors such as fractures, recent stream sinkholes and caverns, and joint-enlarged openings. Hence the permeability is not uniform in all directions.

PURPOSE OF INVESTIGATION

The Bureau of Land Management (1980) prepared a technical report on the hydrology of different aquifers of the Black Hills area. They simulated three-dimensional ground-water flow in the multi-layered aquifer system. The numerical model results showed irregularly shaped contours of drawdown when water was pumped from the Madison aquifer at the proposed ETSI Niobrara well field, in Niobrara County, Wyoming. This created an interest to mathematically analyze the cone of depression in such anisotropic aquifers.

ANALYSIS OF THE CONE OF DEPRESSION

Four methods of cone of depression analysis available in the literature were given by Papadopulos (1965), Hantush and Thomas (1966), Glover and Moody (1976), and Walton (1985). Each method has its own theory for developing a mathematical equation for the drawdown. Extensive theoretical derivation of the equation of drawdown for each method is available in the dissertation by Dayananda (1993).

Each method mentioned above assumes that the anisotropic transmissivity tensor directions coincide with the Cartesian coordinate axes X and Y. That is, if the maximum transmissivity value lies along the X axis or at an angle ϕ, then the corresponding minimum transmissivity value lies along the Y axis, or at an angle of $90^0+\phi$ (Figure 1). If the pumping well is represented by the origin of the Cartesian coordinate system, then the water pumped from the pumping well creates an elliptical distribution of drawdown with the major axis lying along the X axis and the minor axis along the Y axis (Figure 2). The value of drawdown in each direction could be calculated to give an oval configuration of drawdown. If the values of transmissivity in each direction of the Cartesian coordinate system are known, then the cone of depression around the pumping well creates irregularly shaped drawdown contours.

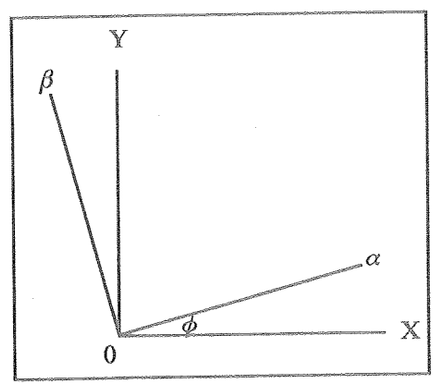

Figure 1. Orientation of principal directions of anisotrophy with respect to Cartesian coordinate axes. Note that the major axis α is perpendicualr to the minor axis β.

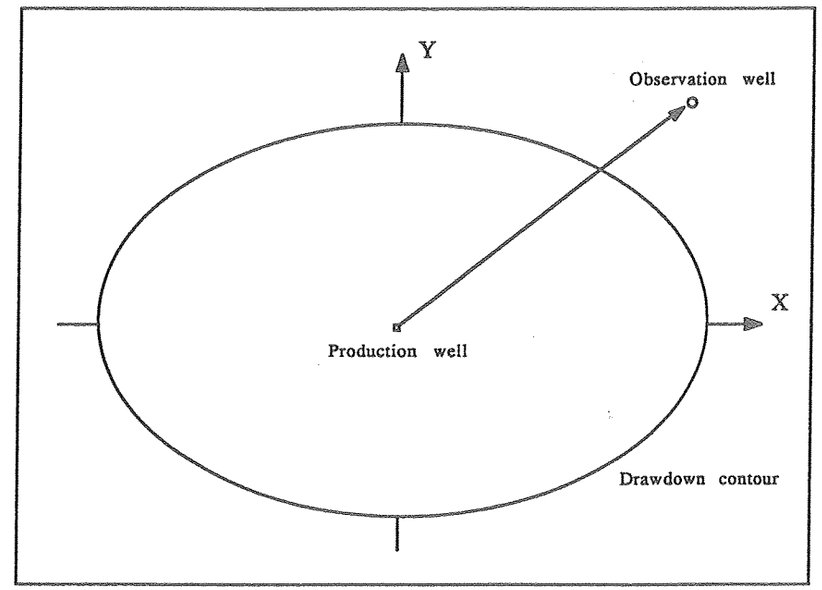

Figure 2. Wells in a uniformly porous nonleaky artesian aquifer with transmissivity varying in two directions (Walton, 1985).

HYPOTHETICAL PUMPING TEST

The above four methods of analysis were studied by assuming a hypothetical pumping test for the anisotropic Madison aquifer. Based on the published values of transmissivity and storage coefficient for the Madison aquifer, the following values were assumed:

Transmissivity in the X direction = T_x = 500 ft^2/day.

Storage Coefficient = S = 0.00005.

The transmissivity in the Y direction was assumed to be one order of magnitude smaller than that in the X direction. The pumping was assumed to continue for one year at a constant pumping rate of 500 gal/min (gpm).

The above values were substituted into the drawdown equation for each method of analysis of the cone of depression. A series of concentric, elliptical distributions of drawdown around a pumping well was obtained (Figure 3).

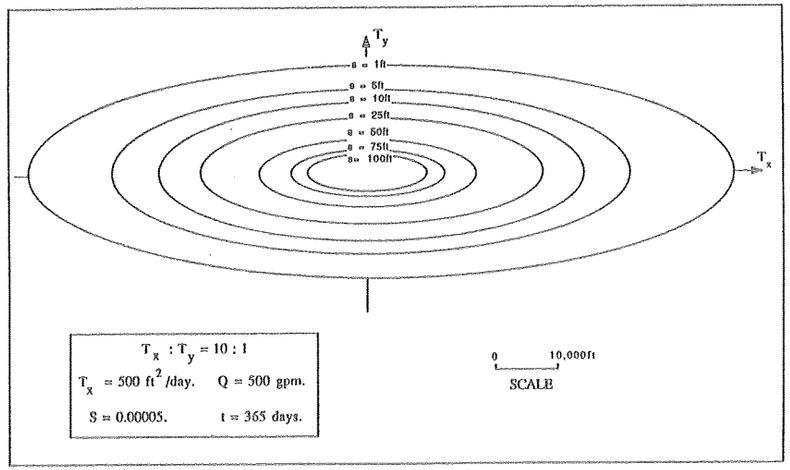

Figure 3. Distribution of drawdown around a pumping well in an anisotropic aquifer.

ANALYTICAL MODEL

A computer program was written in FORTRAN for the Hantush and Thomas (1966) method to calculate the major and minor axis values of an ellipse for a particular drawdown value. A subroutine was written to calculate the "inverse well function" based on Theis's (1935) well function values. This subroutine makes use of the polynomial approximations for the exponential integral and related functions (Gautschi and Cahill, 1964) to facilitate the programming of W(u) series in the Theis equation.

The above polynomial approximation was modified to calculate the values of u from the known values of W(u). The modification requires an initial estimate of the value of u and iterates successively to arrive at the exact value of u. The Newton-Raphson method of iteration was used in the subroutine. The iteration process stops when a specified tolerance limit (closure criterion) is reached. The value of u converges more easily (with fewer iterations) if the initial estimate of u is close enough to the exact value. For the listing of the computer program, the readers are referred to Dayananda (1993).

RESULTS

Each method was analyzed by using hypothetical pumping test values for the Madison aquifer. The methods developed by Hantush and Thomas (1966), Glover and Moody (1976), and Walton (1985) yielded similar drawdown distributions, while that developed by Papadopulos (1965) yielded a slightly different drawdown distribution. However, based on the derivation of the drawdown equation and the ability of the equation to modify any specific problems, the Hantush and Thomas (1966) method was found to be more appropriate.

Figure 3 shows the distribution of drawdown around a pumping well in an anisotropic aquifer, as calculated by using the Hantush and Thomas (1966) method. The results from the analytical model also showed a drawdown distribution identical to the mathematical analysis of the Hantush and Thomas method. It is seen from the figure that the radius of influence extends to 230,000 feet along the major transmissivity tensor (X-axis) direction and 70,000 feet along the minor transmissivity tensor (Y-axis) direction.

CONCLUSION

Based on the assumption that the major transmissivity tensor is perpendicular to the minor transmissivity tensor, the cone of depression created around a pumping well represents an ellipse with the major and minor axes lying along the principal transmissivity tensor directions. A series of concentric elliptical drawdown distributions can be obtained if the transmissivity is known along different directions. The corresponding contour lines of drawdown along different directions can be joined to obtain irregularly shaped contours of drawdown around a pumping well in an anisotropic aquifer.

Although the analytical model developed for the distribution of drawdown is in two different directions (along the Cartesian coordinate axes), future research could be developed to compute the drawdown along different directions. To do this, characterization of the transmissivity along different directions is necessary.

REFERENCES CITED

Bureau of Land Management. 1980. *Well-field hydrology*. Technical Report on the ETSI coal slurry pipeline project, Woodward-Clyde Consultants.

Dayananda, D.R. 1993. Anisotropic and vadose models for aquifers in the Black Hills, South Dakota. Ph.D. dissertation, South Dakota School of Mines and Technology, Rapid City.

Gautschi, W., and W.F. Cahill. 1964. Exponential integrals and related functions. IN Abramowitz, M., and I.A. Stegun (eds.), *Handbook of mathematical functions with formulas, graphs, and mathematical tables*. U.S. Department of Commerce, National Bureau of Standards Applied Mathematical Series, 55:227-251.

Glover, E.R., and T.W. Moody. 1976. Drawdown due to pumping in an anisotropic aquifer. *Water Resources Bulletin* 12(5):941-950.

Hantush, S.M., and G.R. Thomas. 1966. A method for analyzing a drawdown test in anisotropic aquifer. *Water Resources Research* 2:281-285.

Papadopulos, I.S. 1965. Non-steady flow to a well in an infinite anisotropic aquifer. *Proceedings of the Dubrovnik symposium on the hydrology of fractured rocks*, International Association of Scientific Hydrology, 21-31.

Rahn, P.H. 1992. Aquifer hydraulics in a deep confined Cretaceous aquifer at Wall, South Dakota. *Proceedings of Association of Engineering Geologists*, Annual meeting, Los Angeles, 409-418.

Theis, C.V. 1935. The relation between the lowering of peizometric surface and the rate and duration of discharge of a well using ground-water storage. *Transactions of American Geophysics Union*, 16th Annual meeting, part 2.

Walton, W.C. 1985. *Practical aspects of ground-water modeling*. Second ed. Pages 152-155. National Water Well Association,